Fill in the blanks by writing the **symbolic** translations. Do not use $\sim$, $\cup$, $\cap$, $-$ or ^{C} in your answers. **SCORE: _____ / 4 PTS**

(Assume that x is a particular element, and A and B are subsets of universal set U.)

eg. $x \in A \cup B^C$ if and only if $\underline{x \in A \quad \vee \quad x \notin B}$

[a] $x \in A^C - B^C$ if and only if $\underline{x \notin A \quad \wedge \quad x \in B}$

1 POINT EACH

[b] $x \notin A \cap B$ if and only if $\underline{x \notin A \quad \vee \quad x \notin B}$

[c] $B^C \not\subset A$ if and only if $\underline{(\exists x \in U: \quad x \notin B \quad \wedge \quad x \notin A) \quad \vee \quad (\forall x \in A: \quad x \notin B)}$

NOTE: $\subset$ means "is a proper subset of".

Prove that for all sets A and B, if $A \cap B = A$, then $A \cup B = B$. SCORE: _____ / 6 PTS

<u>NOTE: You may NOT use Theorems 6.2.2, 6.2.3, Proposition 6.2.6, or any exercises from the textbook as justification.</u>

<u>You may use Theorem 6.2.1 as justification.</u>

Proof: Let A and B be particular but arbitrarily chosen sets such that $A \cap B = A$

1 $B \subseteq A \cup B$ by Theorem 6.2.1.2b

Let x be a particular but arbitrarily chosen element of $A \cup B$

So, $x \in A$ or $x \in B$ by definition of $\cup$

1/2 POINT EACH UNLESS OTHERWISE NOTED

Case 1: $x \in A$

1

So, $x \in A \cap B$ (since $A \cap B = A$)

So, $x \in B$ by definition of $\cap$

Case 2: $x \in B$

So, $x \in B$ (by division into cases)

So, $A \cup B \subseteq B$ by definition of $\subseteq$

Therefore, $A \cup B = B$ by definition of $=$

Prove that for all sets A and B, $A-B$ and $A\cap B$ are disjoint in two ways. **SCORE: _____ / 10 PTS**

[a] without using Theorems 6.2.1, 6.2.2, 6.2.3, Proposition 6.2.6, or any exercises from the textbook as justification (this is the style of proof presented in lecture)

Proof by contradiction:

1 Assume not, that is suppose there are sets A and B such that $A-B$ and $A\cap B$ are not disjoint

So, $(A-B)\cap(A\cap B)\neq\varnothing$ by definition of disjoint

So, there exists an element $x\in(A-B)\cap(A\cap B)$ by definition of $\varnothing$

$x\in A-B$ and $x\in A\cap B$ by definition of $\cap$

$x\in A$ and $x\notin B$ by definition of $-$

$x\in A$ and $x\in B$ by definition of $\cap$

1 So, $x\notin B$ and $x\in B$ (contradiction)

Therefore, by contradiction, for all sets A and B, $A-B$ and $A\cap B$ are disjoint.

BOTH PARTS:

$\frac{1}{2}$ POINT EACH

UNLESS OTHERWISE

NOTED

[b] using only Theorem 6.2.2 (this is the style of proof in examples 6.3.2 and 6.3.3 in your textbook)
NOTE: Remember to use the commutative and associative laws properly

$(A-B)\cap(A\cap B)$

$=(A\cap B^C)\cap(A\cap B)$ Set Difference Law

$=((A\cap B^C)\cap A)\cap B$ Associative Law

$=(A\cap(A\cap B^C))\cap B$ Commutative Law

$=((A\cap A)\cap B^C)\cap B$ Associative Law

$=(A\cap B^C)\cap B$ Idempotent Law

$=A\cap(B^C\cap B)$ Associative Law

$=A\cap(B\cap B^C)$ Commutative Law

$=A\cap\varnothing$ Complement Law

$=\varnothing$ Universal Bound Laws

MUST HAVE PROPER JUSTIFICATION ON EACH LINE TO EARN ANY POINTS FOR THAT LINE

Let B be a Boolean algebra with operations $+$ and $\cdot$, **SCORE: _____ / 6 PTS**

and let a and b be particular but arbitrarily chosen elements of B. Prove the following statements (which are **NOT** related to each other).

NOTE: Along with the definition of a Boolean algebra, you may use Theorem 6.4.1 as justification without proving it.
Remember to use the commutative and associative laws properly

[a] If $a \cdot b = 1$, then $a = 1$

a

$= a \cdot 1$ Identity Law $\frac{1}{2}$

1 $= a \cdot (a \cdot b)$ Given

$= (a \cdot a) \cdot b$ Associative Law $\frac{1}{2}$

1 $= a \cdot b$ Idempotent Law

$= 1$ Given $\frac{1}{2}$

[b] If $a + b = a$, then $a \cdot b = b$

$a \cdot b$

1 $= (a + b) \cdot b$ Given

$= (b + a) \cdot b$ Commutative Law $\frac{1}{2}$

1 $= b$ Absorption Law

MUST HAVE PROPER JUSTIFICATION ON EACH LINE TO EARN ANY POINTS FOR THAT LINE

One of the following statements is true and one is false. **SCORE: _____ / 9 PTS**

State clearly which statement is false, show **clearly** that it is false, then write a **formal proof** for the true statement.

NOTE: You may NOT use Theorems 6.2.2, 6.2.3, Proposition 6.2.6, or any exercises from the textbook as justification. You may use Theorem 6.2.1 as justification.

HINT: You may use exactly ONE of the assigned homework exercises as a justification in your proof without proving it.

[a] For all sets A and B, $\wp(A-B) \subseteq \wp(A) - \wp(B)$

[b] For all sets A and B, $\wp(A \cap B) = \wp(A) \cap \wp(B)$

[a] is false.

If $A = B = \varnothing$,

then $\wp(A-B) = \wp(\varnothing - \varnothing) = \wp(\varnothing) = \{\varnothing\}$, 1

but $\wp(A) - \wp(B) = \wp(\varnothing) - \wp(\varnothing) = \varnothing$

$\{\varnothing\} \not\subseteq \varnothing$

[b] is true.

Let A and B be particular but arbitrarily chosen sets

Let X be a particular but arbitrarily chosen element of $\wp(A \cap B)$

So, $X \subseteq A \cap B$ by definition of $\wp$

and $A \cap B \subseteq A$ and $A \cap B \subseteq B$ by Theorem 6.2.1.1

So, $X \subseteq A$ and $X \subseteq B$ by Theorem 6.2.1.3 (transitivity of $\subseteq$)

So, $X \in \wp(A)$ and $X \in \wp(B)$ by definition of $\wp$

So, $X \in \wp(A) \cap \wp(B)$ by definition of $\cap$

So, $\wp(A \cap B) \subseteq \wp(A) \cap \wp(B)$ by definition of $\subseteq$

Let X be a particular but arbitrarily chosen element of $\wp(A) \cap \wp(B)$

So, $X \in \wp(A)$ and $X \in \wp(B)$ by definition of $\cap$

So, $X \subseteq A$ and $X \subseteq B$ by definition of $\wp$

1 So, $X \subseteq A \cap B$ by 6.2 Exercise 16

So, $X \in \wp(A \cap B)$ by definition of $\wp$

So, $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$ by definition of $\subseteq$

So, $\wp(A \cap B) = \wp(A) \cap \wp(B)$ by definition of $=$

$\frac{1}{2}$ POINT EACH UNLESS OTHERWISE NOTED